AN ALGORITHMIC METHOD FOR REAL-TIME 3-D MEASUREMENT

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ABSTRACT

Photogrammetric methods will increasingly be used for real-time applications. A typical requirement is the continuous 3-D measurement of target locations which arise from three or more cameras at 0.02 ms. per measurement. In this situation user interaction with algorithms and hardware will be relatively unimportant and a range of new issues will assume greater significance. For instance, if 100-1000 target locations must be measured, then the computational effort must be minimised and if possible completely predictable. Furthermore, the external parameters of the cameras must be checked and, if necessary, adjusted at the same time as the 3-D co-ordinates are measured, while the internal parameters may be adjusted more slowly. Hence, under these conditions, the characteristics of the currently available algorithms and the way in which they are applied must be studied.

This paper describes a methodology for solving collinearity equations based on iterative least squares estimation. Unlike the traditional bundle adjustment which solves for the unknown co-ordinates of object targets and camera parameters *simultaneously*, a solution for least squares estimation is developed which *separates* the parameters into two different groups, one for camera parameters, and the other for the co-ordinates of object points. Each group of parameters is adjusted individually with the other group fixed. While conventionally this process may be carried out just once for a variety of purposes, by repeating this process both sets of parameters are gradually refined. Because the same functional model is used in the two steps and the process is still a conventional least squares optimisation, the final result is the same as that obtained using the usual bundle adjustment but with a considerable time and storage saving. The full covariance matrix is not available, but it will not always be necessary in real-time systems and it can always be computed if required.

1. INTRODUCTION

In close range photogrammetry multiple CCD cameras are used to capture images of the targeted object from different viewpoints. Based on the geometric perspective principle, a set of so called collinearity equations can be derived to establish the relationships between 2-D observations on the camera image planes and 3-D co-ordinates of object targets. By solving the collinearity equations the 3-D co-ordinates of these targets can be estimated. Three major steps are normally needed for this procedure: (i) 2-D image data acquisition and target location; (ii) target matching between different cameras; and (iii) least squares estimation of the unknown parameters of the functional model. Using powerful processors or hardware real-time target location can be realised. Various approaches to target matching are possible such as using epipolar lines and epipolar planes (2-D and 3-D matching). However, solving collinearity equations is still a considerable time consuming procedure. It is not appropriate within the confines of this paper to give a full review of the historical development of least squares optimisation methods so some references and highlights are given which are pertinent to the contents of this paper. The principles of simultaneous least squares adjustment are well known (Mikhail, 1981; Cooper, 1987). It is clear that this method provides the *de facto* standard for the *output* from an adjustment. However, the requirement for large matrix inversions places large demands on storage and computing power. To avoid this a sequential adjustment may be used as a means of providing fast updates for a few parameters while not requiring a full matrix inversion (Shortis, 1980; Gruen, 1985). For most true real-time applications the direct linear transform (DLT) has been used but it does not provide the highest accuracy due to its modelling deficiencies and the reliance on accurately measured control points for camera parameter estimation (Marzan, 1975; Karara, 1980). For situations where interior and exterior camera parameters are known a direct spatial intersection may be used (Granshaw, 1980; Shmutter, 1974). Because each of these methods have deficiencies research is necessary to find an alternative fast, robust and flexible solution.

This paper discusses a two step separated least squares adjustment. It can be shown that this method gives the same results as the simultaneous bundle adjustment but with a significant decrease in storage requirements and computational time. While this method may not be new, to the authors knowledge this is the first time the method has been discussed in the context of real-time 3-D measurement. For example: Shmutter & Perlmuter (1974) discussed the use of iterations of the process of resection followed by intersection to save computer storage space. In this case the functional model was not the same in the two steps hence the results could not be the same as for a simultaneous bundle adjustment; Miles (1963) discussed the solution of normal equations by an iterative process where submatrices representing part of the unknown parameters were solved separately. This was done to save computing storage requirements; and Hill et al (1995) described a two stage iterative solution for image interpretation based on a point distribution model.

2. THEORETICAL BACKGROUND FOR ITERATIVE LEAST SQUARES ESTIMATION

Least squares estimation is an efficient method dealing with redundant measurement containing random errors of normal distribution. It has being widely used in control surveying and photogrammetry to evaluate unknown parameters when the measured elements are more in number than the minimum needed for a unique solution. In this section the normal least squares method is briefly reviewed and an iterative separated least squares method is introduced by a simple example.

2.1 Simultaneous least squares estimation

The following example illustrates the use of least squares estimation for plane positioning of point **P** by measured distances to base stations. Fig. 2.1 illustrates the measurement network.

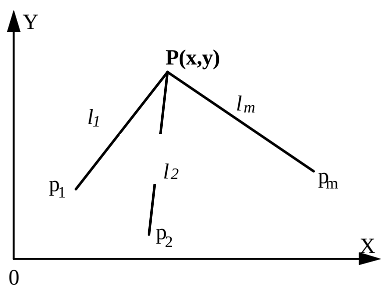

Fig. 2.1 Plane positioning by measured distances to base stations

Eq. 2.1 defines the relationship between the measured elements $l = [\ l_1 , l_2 , \ldots , l_m\ ]$ and the unknown parameters ***x*** and ***y***.

$$f_n = ((x - x_n)^2 + (y - y_n)^2)^{\frac{1}{2}} - l_n = 0 \quad (n = 1, 2, \ldots , m) \qquad 2.1$$

where ***x*** and ***y***, the co-ordinates of the plane point **P**, are two unknown parameters to be estimated, $\boldsymbol{x_n}$ and $\boldsymbol{y_n}$ are co-ordinates of base stations whose value are known and l_n are measured distances from the point **P** to the base stations. A minimum of two base stations will give a unique solution for ***x*** and ***y***. For accurate positioning, more base stations may be used. The least squares method is chosen to estimate the best solution. Linearizing Eq. 2.1 by Taylor series expansion to the first order accuracy gives

$$\frac{\partial f_n}{\partial x}\Delta x + \frac{\partial f_n}{\partial y}\Delta y = -f_n \qquad (n = 1, 2, \ldots , m) \qquad 2.2$$

Expressing Eq. 2.2 in matrix form gives the linearized observation equation

$$A\Delta = b + v \qquad 2.3$$

where ***A*** is the Jacobian matrix of size $m \times 2$, $\boldsymbol{\Delta}$ is a vector of corrections of the unknown parameters, ***b*** is a vector corresponding to the left hand side of Eq. 2.2 and ***v*** is a vector of residuals of the measured elements. Associated with the vector of measured elements ***l*** is a covariance matrix $\boldsymbol{\Sigma}_{ll}$ which gives the stochastic model of the observations. The cofactor matrix $Q_{ll} = \sigma_0^{-2}\Sigma_{ll}$ is used to derive the cofactor matrix of the unknown parameters $Q_{\Delta\Delta}$. Least squares methods usually estimate all the unknown parameters *simultaneously.* This leads to the well known formula of least squares solution

$$\Delta = (A^t W A)^{-1} A^t W b \qquad 2.4$$

where $W=Q_{ll}^{-1}$, is the weight matrix of measured elements. The cofactor matrix the unknown parameters is given by

$$Q_{\Delta\Delta} = (A^t W A)^{-1} \qquad 2.5$$

which gives the accuracy evaluation of unknown parameters obtained from Eq. 2.4. By inversion of the matrix (A^tWA) and some simple matrix calculation, all the unknown parameters can be solved *simultaneously.* This traditional least squares method has been proved to be very efficient in many applications and well justified (Cooper, 1987). A problem may arise when large numbers of parameters need to be estimated by the traditional least squares method in some real applications since inverting large matrices is expensive in terms of speed and memory requirements. In this case the following method may be more suitable.

2.2 Iterative separated least squares estimation

This least squares method estimates unknown parameters one by one. When estimating one selected parameter, all other parameters are considered constant. Another parameter is then selected and the process repeated. After all parameters have been estimated further iterations of the complete process are used until the solution converges satisfactorily. In this example, only two parameters are to be estimated. When estimating the unknown parameter ***x***, the other parameter ***y*** is considered to be constant. When estimating ***y***, ***x*** is considered constant. To fulfil this operation Eq. 2.3 is rearranged as

$$A_1 \Delta x + A_2 \Delta y = b + v \qquad 2.6$$

where

$$A_1 = [\frac{\partial f_1}{\partial x}, \frac{\partial f_2}{\partial x}, \ldots, \frac{\partial f_m}{\partial x}]^t, A_2 = [\frac{\partial f_1}{\partial y}, \frac{\partial f_2}{\partial y}, \ldots, \frac{\partial f_m}{\partial y}]^t$$

are column vectors of the Jacobian matrix ***A***. To estimate the unknown parameter ***x***, ***y*** is considered constant. So $\boldsymbol{A_2} = 0$. In this case, Eq. 2.6 becomes

$$A_1 \Delta x = b + v \qquad 2.7$$

By the criteria of least squares estimation

$$\Delta x = (A_1{}^t W A_1)^{-1} A_1{}^t W b \qquad 2.8$$

Obviously, the dimension of the matrix ($\boldsymbol{A_1{}^t W A_1}$) is 1×1, therefore

$$\Delta x = (\sum_{i=1}^{m} w_i (\frac{\partial f_i}{\partial x})^2)^{-1} (\sum_{i=1}^{m} w_i (\frac{\partial f_i}{\partial x})(-f_i)) \qquad 2.9$$

Similarly,

$$\Delta y = (\sum_{i=1}^{m} w_i (\frac{\partial f_i}{\partial y})^2)^{-1} (\sum_{i=1}^{m} w_i (\frac{\partial f_i}{\partial y})(-f_i)) \qquad 2.10$$

In this way, the unknown parameters ***x*** and ***y*** can be solved separately and iteratively, and inversion of the matrix (A^tWA) is avoided. The accuracy of estimated parameters ***x*** and ***y*** are given approximately by

$$Q_x = (A_1{}^t W A_1)^{-1} \qquad 2.11$$

$$Q_y = (A_2{}^t W A_2)^{-1} \qquad 2.12$$

Table 2.1 gives a comparison of the results between the simultaneous least squares method and the iterative separated least squares method for this plane positioning example with various numbers (*m = 10, 100, 1000*) of base stations surrounding the point **P**(x,y) whose position is to be estimated. Distances l_i (i = 1, 2, ... , m) from point **P** to each base station are measured. The *a priori* standard deviation of each measured distance is given by $s_i = s_o \bar{l}_i$, in which σ_o is the reference variance which is taken to be 0.1 m. for l_o = 100 m. In this simulation test, $(\bar{x}, \bar{y})$ = (600.0, 500.0) is the true position of the point **P**. (590, 510) is selected as the starting point for both methods. The last two rows of Table 2.1 shows the standard errors of ***x*** and ***y*** calculated from Eq. 2.11, Eq. 2.12 and Eq. 2.5.

No. BS	10		100		1000	
Iterations	x	y	x	y	x	y
0	590.00000	510.00000	590.00000	510.00000	590.00000	510.00000
1	599.91478	499.99835	599.85208	500.02114	600.01219	499.99928
2	600.03396	499.99296	600.02077	500.01939	600.00379	499.99861
3	600.03412	499.99295	600.02079	-----	600.00378	-----
SLS	600.03412	499.99295	600.02079	500.01939	600.00378	499.99861
ISLS RMS	0.08056	0.06587	0.03087	0.02528	0.00957	0.00790
SLS RMS	0.08545	0.06986	0.03103	0.02541	0.00958	0.00791

Table 2.1 Comparison between Simultaneous Least Squares (SLS) and Iterative Separated Least Squares estimation (ISLS)

It can be seen from Table 1 that the iterative least squares method gives exactly the same results as the simultaneous least squares method after two or three iterations for this example and the approximately estimated standard errors of ***x*** and ***y*** from Eq. 2.11 and Eq. 2.12 are comparable with that calculated from Eq. 2.5. Although this method is demonstrated for a specific simple example, the results hold for other more complex situations that require least squares estimation. The computational expenses of the two methods are not compared in this example since only two unknown parameters are involved. If many unknown parameters, say hundreds or even thousands, are to be estimated and the observation equations have the special structure that is typical in photogrammetry, the iterative separated least squares method will give a significant advantage in terms of speed and memory requirements. Furthermore, although the unknown parameters may be estimated one by one, they may also be estimated in groups.

3. THE APPLICATION OF SEPARATED LEAST SQUARES ESTIMATION IN PHOTOGRAMMETRY

3.1 Bundle adjustment

The bundle adjustment is a well known and powerful analytical method for the determination of 3-D co-ordinates where optimum results and statistical information are required. The procedure of bundle solution is briefly reviewed here. If $\boldsymbol{N_c}$ cameras are used to measure $\boldsymbol{N_p}$ object points and the ***jth*** object point (X_j, Y_j, Z_j) is imaged on the ***ith*** camera as a image point (x_{ij}, y_{ij}), 2$\boldsymbol{N_c N_p}$ equations can be constructed by the collinearity conditions. The well known collinearity equations are expressed as

$$\begin{aligned} x_{ij} &= -f_i \frac{m_{i11}(X_j - X_{Li}) + m_{i12}(Y_j - Y_{Li}) + m_{i13}(Z_j - Z_{Li})}{m_{i31}(X_j - X_{Li}) + m_{i32}(Y_j - Y_{Li}) + m_{i33}(Z_j - Z_{Li})} \\ y_{ij} &= -f_i \frac{m_{i21}(X_j - X_{Li}) + m_{i22}(Y_j - Y_{Li}) + m_{i23}(Z_j - Z_{Li})}{m_{i31}(X_j - X_{Li}) + m_{i32}(Y_j - Y_{Li}) + m_{i33}(Z_j - Z_{Li})} \end{aligned} \qquad 3.1$$

$$(i = 1, 2, \dots , \boldsymbol{N_c} \qquad j = 1, 2, \dots , \boldsymbol{N_p})$$

where X_j, Y_j, Z_j are the 3-D co-ordinates of the ***jth*** object point , x_{ij}, y_{ij} are the 2-D co-ordinates of its image on the ***ith*** camera, X_{Li}, Y_{Li}, Z_{Li} are the perspective centre position of the ***ith*** camera, and m_{imn} are the rotation coefficients derived from w_i, j_i, k_i of the ***ith*** camera. (3$\boldsymbol{N_p}$+6$\boldsymbol{N_c}$) unknown parameters have to be solved. Usually the number of equations 2$\boldsymbol{N_c N_p}$ (2$N_c N_p$+7 when datum deficiencies are considered) is much larger than the number of unknown parameters (3$\boldsymbol{N_p}$+6$\boldsymbol{N_c}$). So these unknown parameters can be estimated by the least squares method and the 3-D co-ordinates of each object point can then be obtained. The traditional bundle adjustment estimates all unknown parameters simultaneously. This means the dimensions of the coefficient matrix ($\boldsymbol{A^t W A}$) of the linearized observation equations will be (3$\boldsymbol{N_p}$+6$\boldsymbol{N_c}$)×(3$\boldsymbol{N_p}$+6$\boldsymbol{N_c}$). When the number of object points and/or the number of cameras is very large, the computational and storage expense of inverting the matrix ($\boldsymbol{A^t W A}$) will be considerable. Even if partitioning of the coefficient matrix ($A^t WA$) is considered (Granshaw, 1980), inverting a matrix of 3$\boldsymbol{N_p}$×3$\boldsymbol{N_p}$ or 6$\boldsymbol{N_c}$×6$\boldsymbol{N_c}$ is still time consuming. Table 3.1 illustrates the computational expense of a typical Bundle Adjustment operated on *SUN SPARC Classic* workstation.

Number of targets	50	100	150	200	250	300	350	400
Computational time (s)	11	66	214	521	997	1690	3488	3967

Table 3.1 Computational expense of Bundle Adjustment (Number of Cameras $\boldsymbol{N_c}$ = 5)

3.2 Two step separated adjustment

When the iterative least squares method is used to deal with the collinearity equations in photogrammetry, all unknown parameters can be estimated one by one from the 3-D co-ordinates of each object point to the parameters of each camera. In this case, all the object points are independent when the camera parameters are considered constant and all cameras are independent when the object points are fixed. This can clearly be seen from the structure of the Jacobian matrix ***A*** (Fig. 3.1a) since there is no rank deficiency of the linearized observation equations. It is convenient to divide all unknown parameters into two groups, one for the 3-D co-ordinates of object points and the other for the camera parameters, i.e., $\boldsymbol{x}$ = $(\boldsymbol{x}_{op}, \boldsymbol{x}_{cp})^t$, where the subscripts *op* and *cp* refer to 3-D co-ordinates of object points and camera parameters respectively. This technique may be termed a *two step separated adjustment* in photogrammetry. The two grouped unknown parameters $\boldsymbol{x}_{op}$ *and* $\boldsymbol{x}_{cp}$ are expressed as $\boldsymbol{x}_{op} = (X_1, Y_1, Z_1, X_2, Y_2, Z_2, \dots, X_{Np}, Y_{Np}, Z_{Np})^t$ $\boldsymbol{x}_{cp} = (X_{L1}, Y_{L1}, Z_{L1}, w_1, j_1, k_1, X_{L2}, Y_{L2}, Z_{L2}, w_2, j_2, k_2, \dots, X_{LNc}, Y_{LNc}, Z_{LNc}, w_{Nc}, j_{Nc}, k_{Nc})^t$. In this case, the Jacobian matrix ***A*** is separated into two parts $\boldsymbol{A}_{op}$ and $\boldsymbol{A}_{cp}$, the linearized observation equations will be expressed as

$$\begin{bmatrix} A_{op} & A_{cp} \end{bmatrix} \begin{bmatrix} \Delta x_{cp} \\ \Delta x_{cp} \end{bmatrix} = b + v \qquad 3.2$$

where $\boldsymbol{A}_{op}$ and $\boldsymbol{A}_{cp}$ are submatrices of Jacobian matrix $\boldsymbol{A}$ which refer to the partial differentials of the functional model with respect to $\boldsymbol{x}_{op}$ and $\boldsymbol{x}_{cp}$ respectively. Fig. 3.1a illustrates the structure of the Jacobian matrix $\boldsymbol{A}$ with the size of $2N_cN_p$´$(3N_p+6N_c)$, the left hand section is $\boldsymbol{A}_{op}$ and right hand section is $\boldsymbol{A}_{cp}$. Each small block in $\boldsymbol{A}_{op}$ indicates a *2´3* submatrix and the big block in $\boldsymbol{A}_{cp}$ indicates a $2N_p$´6 submatrix. (N_p is the number of object points and N_c is the number of cameras).

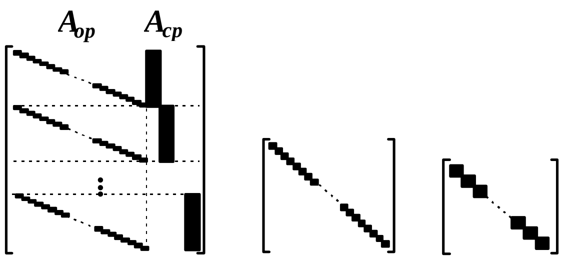

Fig. 3.1 (a) (b) (c)

(a) The structure of the Jacobian matrix $\boldsymbol{A}$ of the linearized collinearity equations. (b) The structure of the coefficient matrix $\boldsymbol{A}_{op}^{t}W\boldsymbol{A}_{op}$. (c) The structure of the coefficient matrix $\boldsymbol{A}_{cp}^{t}W\boldsymbol{A}_{cp}$

The basic principle of this two step separated adjustment method is to estimate the unknown parameters $\boldsymbol{x}_{op}$ and $\boldsymbol{x}_{cp}$ separately and iteratively. When estimating 3-D co-ordinates of object points $\boldsymbol{x}_{op}$, camera parameters $\boldsymbol{x}_{cp}$ are considered constant. When estimating camera parameters $\boldsymbol{x}_{cp}$, 3-D co-ordinates of object points $\boldsymbol{x}_{op}$ are considered constant. This two step separated procedure is discussed in detail as follows.

3.2.1 Adjust 3-D co-ordinates of object points with cameras fixed

If the camera parameters $\boldsymbol{x}_{cp}$ are supposed to be constant, all elements of matrix $\boldsymbol{A}_{cp}$ will be zero. Eq. 3.3 is simplified to

$$A_{op}\Delta x_{op} = b_{op} + v_{op} \qquad 3.3$$

in which b_{op} and v_{op} are column vectors corresponding to the right hand sides of Eq. 3.3 respectively. By the criteria of least squares estimation, the corrections of object points $\Delta\boldsymbol{x}_{op}$ is given by

$$\Delta x_{op} = (A_{op}{}^{t}WA_{op})^{-1}A_{op}{}^{t}Wb_{op} \qquad 3.4$$

where the coefficient matrix ($\boldsymbol{A}_{op}{}^{t}\boldsymbol{W}\boldsymbol{A}_{op}$) is a block diagonal matrix with the size of $3N_p$´$3N_p$ whose structure is shown in Fig. (3.1b). Each small block on the diagonal is a *3´3* submatrix. In this case, the covariance matrix $(\boldsymbol{A}_{op}{}^{t}\boldsymbol{W}\boldsymbol{A}_{op})^{-1}$ of the 3-D co-ordinates of object points will have the same structure as the coefficient matrix ($\boldsymbol{A}_{op}{}^{t}\boldsymbol{W}\boldsymbol{A}_{op}$) and is calculated simply by inverting N_p *3´3* matrices instead of inverting a $3N_p$´$3N_p$ matrix. Since all the object points are independent their corrections can be estimated one by one. The 3-D co-ordinates of all the object points determined from the previous iteration are used as the starting values in this iteration since the collinearity equations are non-linear. After this iteration, these 3-D co-ordinates of object points are updated and refined. They are then used to adjust the camera parameters in the next procedure.

3.2.2 Adjust camera parameters with object points fixed

In this case, the 3-D co-ordinates of object points $\boldsymbol{x}_{op}$ are considered constant. So all elements in matrix $\boldsymbol{A}_{op}$ are zero. Eq. 3.3 becomes

$$A_{cp}\Delta x_{cp} = b_{cp} + v_{cp} \qquad 3.5$$

in which b_{cp} and v_{cp} are column vectors corresponding to the right hand sides of Eq. 3.3 respectively. By the criteria of least squares estimation, the correction of object points $\Delta\boldsymbol{x}_{cp}$ is given by

$$\Delta x_{cp} = (A_{cp}{}^{t}WA_{cp})^{-1}A_{cp}{}^{t}Wb_{cp} \qquad 3.6$$

where the coefficient matrix ($\boldsymbol{A}_{cp}{}^{t}\boldsymbol{W}\boldsymbol{A}_{cp}$) is also a block diagonal matrix with the size of $6N_c$´$6N_c$ whose structure is shown in Fig. 3.1c. Each small block on the diagonal is a *6´6* submatrix. In this case, the covariance matrix $(\boldsymbol{A}_{cp}{}^{t}\boldsymbol{W}\boldsymbol{A}_{cp})^{-1}$ of the camera parameters will have the same structure as the coefficient matrix ($\boldsymbol{A}_{cp}{}^{t}\boldsymbol{W}\boldsymbol{A}_{cp}$) and it is calculated simply by inverting N_c *6´6* matrices instead of inverting a $6N_c$´$6N_c$ matrix. Since all the cameras are independent their corrections can also be estimated one by one. The parameters of all the cameras determined by a previous iteration are used as the starting values in this iteration. After this iteration, these camera parameters are updated and refined.

3.3 Discussion.

In practise the two step process continues until the required stopping criteria is met. Simulation tests and practical tests show that this two step separated adjustment can always give the same solution as the traditional simultaneous bundle adjustment after a few tens of iterations even for a very weak network and poor starting values. The test results are given in the next section.

It has been assumed here that the focal length is a constant and all systematic errors introduced by lens distortion or any other sources have been calibrated beforehand, and the measured image co-ordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ are corrected accordingly. If these systematic errors have not been calibrated or are not well calibrated, additional parameters can be included in the collinearity equations as for a self calibrating bundle adjustment. In this case, more unknown parameters (e.g. *14* or more) will be involved in the procedure of camera parameter adjustment. Instead of *6´6*, the size of each small block on the diagonal of the coefficient matrix ($\boldsymbol{A}_{cp}{}^{t}\boldsymbol{W}\boldsymbol{A}_{cp}$) could be *14´14*, but the structure of remains the same, it is still a block diagonal matrix. Alternatively, a third step may be used to adjust the interior camera parameters only with object points and exterior camera parameters fixed. It is well known that the Jacobian matrix $\boldsymbol{A}$ and the coefficient matrix ($\boldsymbol{A}^{t}\boldsymbol{W}\boldsymbol{A}$) of linearized observation equations in photogrammetry are very sparse and very special in structure. There are $3N_p+6N_c$ elements on each row of the Jacobian matrix $\boldsymbol{A}$ and only 9 of them are non-zero. So the sparseness of A is given by

$$S_A = \frac{9}{3N_p + 6N_c} \qquad 3.7$$

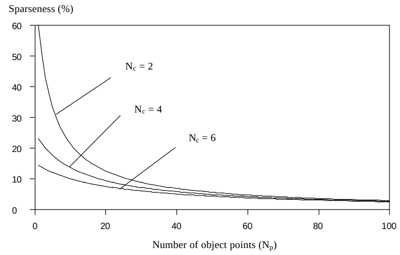

Fig. 3.2 The sparseness of the Jacobian matrix ***A***

Fig. 3.2 illustrates the sparseness of ***A***. When the number of object points is 100, less than 3% of elements in Jacobian matrix ***A*** are non-zero. In this case, inverting the full size matrix of (A^tWA) or two partitioned matrices which is common in the bundle adjustment is not efficient if the full covariance matrix is not necessary. The full covariance matrix may be valuable in some situations to evaluate the whole system, but in many situations (e.g. real-time) the diagonal elements of covariance matrix could be adequate to evaluate the accuracy of estimated 3-D co-ordinates of the object points.

The two step separated adjustment makes full use of the special properties of the Jacobian matrix ***A*** and the principle of iterative least squares estimation. An accuracy evaluation of the estimated 3-D co-ordinates of object points and camera parameters are given approximately by

$$Q_{x_{op}} = (A_{op}{}^t W A_{op})^{-1} \qquad 3.8$$

$$Q_{x_{cp}} = (A_{cp}{}^t W A_{cp})^{-1} \qquad 3.9$$

In some industrial applications, for example real-time monitoring of moving objects, the camera parameters are relatively stable while the 3-D co-ordinates of object points may move frequently. In this case, the object points can be located with good estimates for camera parameters which can also be monitored and if necessary adjusted. In addition real-time 3-D co-ordinate measurement for hundreds of targets can be achieved using inexpensive computers. Care must be taken when the two step separated adjustment is applied in photogrammetry in order to get the same results as the traditional bundle adjustment. The linearized observation equations should be the same for both steps. The objective of the minimisation is the sum of squares of the residuals on the image plane as is usual in the bundle adjustment but is often not the case in many intersection algorithms for example Shmutter & Perlmuter (1974).

4. SIMULATION TESTS

A simulation network was constructed to test the two step separated adjustment method in photogrammetry and compare it with the traditional bundle adjustment. Fig. 4.1 illustrates the configuration of the simulation test network. The object points were randomly distributed in a 400×400×200 mm. box with eight control points on the edges which were used to initialise the camera parameters. The focal length of the cameras was 25 mm. The cameras were uniformly located on a circle with a distance of 2500 mm. to the centre of the box. The 2-D projections of the targets on the image planes were then computed. Approximate camera parameters were calculated using control points with deliberately added errors. Approximate 3-D co-ordinates of the object points were computed using the approximate camera parameter. Both the approximate camera parameters and the 3-D co-ordinates of the object points were then used as the starting values. The results of using the simultaneous adjustment and the separated adjustment were compared.

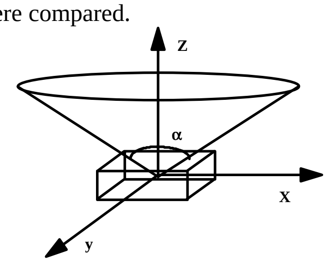

Fig. 4.1 The simulation test network

Table 4.1 shows some simulation test results of the bundle adjustment and the two step separated adjustment for a four camera network. The minimisation of the sum of squares of the residuals (**v^tWv**) on the image plane is the objective of the least squares process. The values of **v^tWv** calculated from both methods were always same (the small differences in the eighth decimal place is caused by the round off of input data), and all residuals on the image planes were the same for the two methods. A further check was made by comparing the difference between 3-D co-ordinates of the object points obtained from both methods after a 3-D transformation. The results indicated no differences to the level of precision used. It can be seen from Table 4.1 that the two step separated adjustment is much faster than the bundle adjustment especially when the number of targets is very large, since this method shows a linear computational expense with the number of targets. To measure 1000 targets for this four camera network, the two step separate adjustment needs only 103 seconds. It should be noted that the two step method iterates more times than the bundle adjustment but for real-time applications only one iteration may be required.

	GAP		**TSSA**	
targets	**Time**(seconds)	**v^tWv**(mm^2)	**Time**(seconds)	**v^tWv**(mm^2)
50	11	0.00026146	5	0.00026144
100	66	0.00053658	11	0.00053662
150	214	0.00082628	17	0.00082622
200	521	0.00105640	22	0.00105626
250	997	0.00126715	27	0.00126712
300	1690	0.00148518	32	0.00148520
350	3488	0.00177046	34	0.00177045
400	3967	0.00206680	41	0.00206678
1000			103	0.00506967

Table 4.1 Number of cameras = 4. (TSSA refers to the two step separated adjustment. GAP, the General Adjustment Program developed at the City University, is a simultaneous least squares estimation program used in survey and/or photogrammetric network adjustment - a typical Bundle Adjustment)

The accuracy of the 3-D co-ordinates of the object points estimated by the two methods are the same since their results are same. In the two step separated adjustment method, the full covariance matrix is not calculated, the accuracy of the 3-D co-ordinates of the object points estimated can only be evaluated approximately by Eq. 3.8. Table 4.2 shows these approximate values and the values calculated from the full covariance matrix with a six camera network. It can be seen that the results are similar especially when the number of targets increases. So the approximately evaluated standard errors appear to be acceptable.

Number of targets	σ_x (mm)		σ_y (mm)		σ_z (mm)	
	GAP	TSSA	GAP	TSSA	GAP	TSSA
50	0.04653	0.04677	0.04644	0.04678	0.05614	0.05738
100	0.04678	0.04689	0.04677	0.04689	0.05696	0.05763
150	0.04685	0.04693	0.04685	0.04693	0.05724	0.05770
200	0.04680	0.04685	0.04680	0.04686	0.05716	0.05752

Table 4.2 Number of cameras = 6, σ_o = 0.001 (mm), α = 90º

It is well known that increasing the number of photographs at each camera station will increase the accuracy of 3-D co-ordinates of the object points measured in photogrammetry. Table 4.3 illustrates the results of the simulation test with six camera stations and 200 targets. When the number of photographs increase, the standard errors for x, y and z decrease and they are inversely proportional to the square root of the number of photographs as reported by Fraser (1992).

Number of photographs	σ_x (mm)	σ_y (mm)	σ_z (mm)
1	0.04686	0.04686	0.05752
2	0.03313	0.03313	0.04067
4	0.02343	0.02343	0.02876
6	0.01913	0.01913	0.02348
8	0.01657	0.01657	0.02034
k	$0.04686\boldsymbol{k}^{-1/2}$	$0.04686\boldsymbol{k}^{-1/2}$	$0.05752\boldsymbol{k}^{-1/2}$

Table 4.3 Number of targets = 200 σ_0 = 0.001 (mm) α = 90º

Changing the network geometry gives different accuracy for estimated 3-D co-ordinates. Table 4.4 and Fig. 4.2 illustrates the influence of network geometry on the accuracy of 3-D co-ordinates by changing the convergent angle α. A large angle will cause the accuracy to worsen in x and y, and get better in z. It can be seen approximately 110º will give the best accuracy for x, y and z (RMS values) and that angles between 100º and 120º are reasonable. The *q*-value is equal to 0.5 in this situation as reported by Fraser (1984).

α(º)	σ_x (mm)	σ_y (mm)	σ_z (mm)	$\bar{\sigma}_{xyz}$(mm)
60	0.04351	0.04351	0.08145	0.05893
80	0.04558	0.04559	0.06330	0.05217
100	0.04826	0.04827	0.05307	0.04992
108	0.04943	0.04944	0.05023	0.04970
110	0.04973	0.04974	0.04960	0.04969
112	0.05004	0.05005	0.04901	0.04970
120	0.05127	0.05128	0.04690	0.04986
140	0.05420	0.05421	0.04317	0.05080
160	0.05642	0.05643	0.04117	0.05184

Table 4.4 Number of targets = 200 Number of cameras = 6 σ_o = 0.001 (mm)

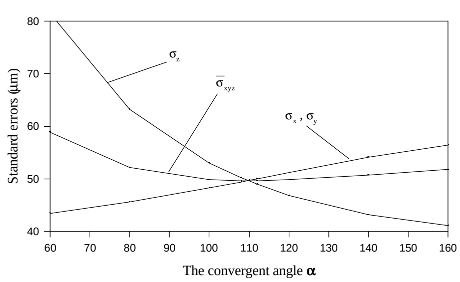

Fig. 4.2 3-D co-ordinate accuracy of different network geometry

5. CONCLUSIONS

In this paper an iterative separated least squares estimation method is introduced and compared with the simultaneous least squares estimation method using a simple example. This method has been applied to the solution of collinearity equations as a two step separated adjustment method. Simulation tests showed that this method gave the same result as the traditional bundle adjustment. The advantages of this method are: (i) it is much faster than the traditional bundle adjustment. The bundle adjustment shows an exponential increase with the number of target, while this iterative method is linear; (ii) less memory is required than the traditional bundle adjustment. With the bundle adjustment, the inversion of the large matrix requires considerable memory space as the number of unknowns increases. With the iterative method, the sizes of the matrices to be inverted are 3x3 and 6x6 no matter how many cameras and targets involved; (iii) it is reliable and robust. Simulation tests show that the convergent property of the separated solution is as good as that of the bundle adjustment; and (iv) it is more flexible than the direct linear transform method, as camera orientations are continually updated and a full functional model of all camera parameters can be included. Further work is undeway to implement this method in a real-time system and to consider other aspects such as: datum problems; further mathematical analysis; and real-time specific issues.

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